(Unit-I) Selection questions of Unit-1. (L.A)

Q.(1) (i) Define vector space with example

(ii) Let $V = \{(a_1, a_2) : a_1, a_2 \in R\} = R^2$;

For $(a_1, a_2) \in V$, $(b_1, b_2) \in V$ and $\alpha \in R$,

Define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 - b_2)$

and $\alpha(a_1, a_2) = (\alpha a_1, \alpha a_2)$

Is V vector space under above operation? Justify your answer.

(iii) $V = R^2$, $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, 0)$

and $\alpha(a_1, a_2) = (\alpha a_1, 0)$

Show that: V, is not a vector space under above mention operations.

(iv) Q.13, Q.14, Q.15, Q.16, Q.17, Q.18 (page-15)

(v) How many matrices are there in the vector space $M_{m \times n}(Z_2)$?

(vi) A vector space must have atleast two elements (T/F)

(vii) Define subspace of a vector space?

(ix) Let V be a vector space and W is a subset of V Then W is a subspace of V iff W satisfy following condition. (a) $0 \in W$,

(b) $x + y \in W$, $\forall\ x, y \in W$.

(c) $\alpha x \in W$, $\alpha \in F$, and $x \in W$.

(x) $W = \left\{ A : A = A^T \right\}_{A = (a_{ij})_{n \times n}}$, Then show that W is subspace of a V.S. $V = M_{n \times n}(F)$, with respect to OMA, OMM.

(xi) $V = M_{n\times n}(\mathbb{R})$ be a v.s. and $W = \{A \in M_{n\times n}(\mathbb{R}) : tr(A) = 0\}$ is a subspace of V with respect to OMA, OMM.

(xii) $V = P[x] = \{p(x) : p(x) \in P[x]\}$ set of all poly.
and $W = P_n[x] = \{p(x) \in P_n(x) : \deg p(x) \le n\}$
Show that W is a subspace of V with respect to standard poly. additn & scalar multiplicatn's.

(xiii) Prove that: Arbitrary intersection of subspaces of a vector space V is a subspace of V.

(xiv) Q.8, Q.10, Q.24. (page-20, 22)

(xv) Let W_1 and W_2 be two subspaces of a v.s. V
(a) Prove that: $W_1 + W_2$ is a subspace of V that contains both W_1 and W_2
(b) Prove that: Any subspace of V that contains both W_1 & W_2 must also contain $W_1 + W_2$.

(xvi) Show that: $2x^3 - 2x^2 + 12x - 6$ is a linear combinatn of $x^3 - 2x^2 - 5x - 3$ and $3x^3 - 5x^2 - 4x - 9$ in $P_3(\mathbb{R})$.

(xvii) Define span of a set.

(xviii) Let V be a v.s. of poly. of degree ≤ 6, which of the following are subspace? Justify your answer.
(a) $W = \{f(x) \in V : \deg f(x) = 4\}$
(b) $W = \{f(x) \in V : f(0) = 1$
(c) $W = \{f(x) \in V : \deg f(x) \le 3\}$

(xix) Consider the vectors $u = (1, 2, 3)$ and $v = (2, 3, 1)$ in $\mathbb{R}^3$
(a) write $w = (1, 3, 8)$ as a linear combination of u and v
(b) Find K so that $w = (1, K, 4)$ is a Linear combination of u & v.

(xx) Show that: $u_1 = (1, 2, 3)$, $u_2 = (0, 1, 2)$ and $u_3 = (0, 0, 1)$ generates $V_3(\mathbb{R})$.

(xxi) Find the condition on a, b, c such that the matrix $\begin{pmatrix} a & -b \\ b & c \end{pmatrix}$ is a linear combination of the matrices $\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$, $\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$.

(xxii) Show that the vectors $(1,1,0)$, $(1,0,1)$, $(0,1,1)$ generate F^3.

(xxiii) Show that: $P_n(F)$ is generated by $\{1, x, \cdots x^n\}$.

(xxiv) Prove that: A subset W of a V.S. V is a subspace of V iff $\text{span}(W) = W$.

(xxv) if S_1, S_2 are subsets of a V.S. V s.t. $S_1 \subseteq S_2$ Then prove that $\text{span}(S_1) \subseteq \text{span}(S_2)$.

(xxvi) Define L.I and L.D.

(xxvii) Show that: The set $S = \{(1,0,0,-1), (0,1,0,-1), (0,0,1,-1), (0,0,0,1)\}$ is linearly independent.

(xxviii) ~~Let V be a V.S. and S~~

(a) Prove that: Super set of L.D. set is L.D.

(b) Prove that: subset of L.I. set is L.I.

(xxix) Let 'V' be a V.S. over field F.

(a) Let u & v be distinct vectors in V. Then prove that: $\{u, v\}$ is L.I iff $\{u+v, u-v\}$ is L.I.

(b) Let u, v, w be distinct vectors in V. Then prove that: $\{u, v, w\}$ is L.I iff $\{u+v, v+w, w+u\}$ is L.I.

(xxx) Define basis for a vector space and dimension of a vector space.

(xxxi) What is dimension of $V = \{0\}$. ~~an empty set of vectors~~

(xxxii) What is dimension of F^n, $M_{n \times m}(F)$, $P_n(F)$, $C(\mathbb{R})$ and $\mathbb{R}(\mathbb{R})$, $F(F)$.

(xxxiii) Show that: $\{x^2+3x-2,\ 2x^2+5x-3,\ -x^2-4x+4\}$ is a basis for $P_2(\mathbb{R})$.

(xxxiv) Let W be a subspace of a FDVS V. Then prove that (a) W is FDVS and $\dim W \le \dim V$.
(b) if $\dim W = \dim V \Rightarrow$ Then $V = W$.

(xxxv) $W = \{(a_1, a_2, a_3, a_4, a_5) \in F^5 : a_1 + a_3 + a_5 = 0,\ a_2 = a_4\}$. Show that: W is a subspace of F^5. Find basis for W and $\dim W$.

(xxxvi) $W = \{A \in M_{n \times n}(\mathbb{R}) : A = A^T\}$ is a subspace of. Find $\dim(W)$.

(xxxvii) Q.2, Q.3, Q.4, Q.5, Q.14, Q.12

Linear-Transformation.

(a)(i) Define Linear transformation.
(ii) Show that: if T is linear then $T(x-y) = T(x) - T(y)$
(iii) Define Null space and Range space of a L.T.
(iv) Prove that: Null space and Range space of a L.T. are Subspace of Domain & Co-domain respectively.
(v) $T : \mathbb{R}^3 \to \mathbb{R}^2$ is defined by
$T(x_1, x_2, x_3) = (x_1 - x_2,\ x_1 + x_2)$, Find Rank(T) & kernel of T.
(vii) What is span of X-axis and xy-plane in F^3.
(viii) Q.2, Q.3, Q.4, Q.5, Q.6, Q.9, Q.15 (page 74-75)

Thm: 2.2, page-68 (ix) Let V and W be V.S., and Let $T : V \to W$ be linear. If $\beta = \{u_1, u_2, \ldots, u_n\}$ is a basis for V, then Prove that: $\text{Range}(T) = R(T) = \text{span}(T(\beta)) = \text{span}\{T(u_1), T(u_2), \ldots, T(u_n)\}$

(page-70) Thm-2.3 (x) State and Prove Rank-Nullity (Dimension theorem or Sylvester Thm)

Thm-2.4 (xi) Let V and W be V.S., and let $T : V \to W$ be linear. Then prove that: T is one-one iff $N(T) = 0$.

Thm 2.5 page-71

(xii) Let V and W be v.s. of equal (finite) DVS. and let $T: V \longrightarrow W$ be Linear. Then show that following statements are equivalent:

(a) T is one-to-one
(b) T is onto.
(c) $\rho(T) = \dim V$

(xiii) Example-11

Let $T: P_2(R) \longrightarrow P_3(R)$ be the L.T. defined by $T(f(x)) = 2f'(x) + \int_0^x 3f(t)\,dt$

Then find $\rho(T)$ and $\eta(T)$.

(xiv) $T: P_2(R) \longrightarrow R^3$ be the L.T. defined by

$$T(a_0 + a_1x + a_2x^2) = (a_0, a_1, a_2)$$

show that: T is one-one and onto.

(xv) Q.14, Q.15

(xvi) Suppose that: $T: R^2 \to R^2$ is linear, $T(1,0) = (1,4)$ & $T(1,1) = (2,5)$, Find $T(2,3)$? Is T one-to-one.

(xvii) Prove that: ∃ a L.T. $T: R^2 \to R^3$ s.t. $T(1,1) = (1,0,2)$ & $T(2,3) = (1,-1,4)$,

Then find $T(8,11)$

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Unit-3 (Selection Questions of L.A)

Q.1(i) Define eigenvectors of a L.O?

(ii) Find all eigenvector of the matrix $\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$.

(iii) Find the eigenvalues and eigenvectors of

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

(iv) Prove that: $\lambda = 0$ is an eigenvalue of the matrix A iff A is singular.

(v) What is algebraic multiplicity of an eigenvalue λ of a L.O T?

(vi) What is Diagonalizable L.O?

(vii) Give an example of a L.O, which has infinite number of eigenvalues

(viii) Let $A \in M_{n\times n}(F)$. Then prove that: a scalar λ is an eigenvalue of A iff $\det(A - \lambda I_n) = 0$.

(ix) Define characteristic poly. of a L.O T.

(Long) (x) Let T be a L.O on $P_2(R)$ defined by $T(f(x)) = f(x) + (x+1) f'(x)$. Test the diagonalizability of T and if T is diagonalizable, find a basis β for V s.t. $[T]_\beta$ is a diagonal matrix.

(xi) Find the eigenvalues of $\begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$.

(xii) Show that: The matrix $\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$ is diagonalizable find ~~a basis β for R²~~ a non singular matrix P and ~~verify~~ find $P^{-1}AP$.

(xiii) Q.2, Q.3, Q.4

(xiv) Let T be a L.O on a V.S V, and let $\lambda_1, \lambda_2, \ldots, \lambda_k$ be distinct eigenvalues of T. If $u_1, u_2, \ldots, u_k$ are eigenvectores of T s.t. λ_i corresponding to u_i $(1 \le i \le k)$, Then prove that: $\{u_1, u_2, \ldots, u_k\}$ is L.I.

(xv) The characteristic poly. of any diagonalizable linear operator splits.

(xvi) Define eigenspace of a L.O T.

(xvii) Let T be a L.O on a FDVS V and let λ be an eigenvalue of T having multiplicity m. Then prove that $1 \leq \dim(E_\lambda) \leq m$.

(xviii) $A = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{pmatrix} \in M_{3\times3}(R)$

Test diagonalizibility.

(xix) $T: P_2(R) \to P_2(R)$ by $T(f(x)) = f'(x)$.

Test diagonalizability?

(xx) Th^m: 5.9 (Page-268)

(xxi) Let T be the L.O on $P_2(R)$ defined by

$T(f(x)) = f(1) + f'(0)x + (f'(0) + f''(0))x^2$.

Test diagonalizability of T and if T is diagonalizable find a basis β for V s.t. $[T]_\beta$ is a diagonal matrix

(xxii) Q. 3 (a), (d), (f) (Page-280)

(xxiii) Define T-invariant subspace of V.

(xxiv) Th^m: 5.21 (Page-314)

(xxv) State & Prove Cayley-Hamilton Th^m.

(xxvi) if $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$, verify Cayley Hamilton Th^m.

(xxvii) Define inner product on a V.S. V

(xxviii) if $x = (1+i, 4)$, $y = (2-3i, 4+5i)$ in C^2.

Compute: $\langle x, y \rangle$ using standard I.P.

(xxix) Let V be an IPS, for any $x, y, z \in V$

Then prove that: if $\langle x, y\rangle = \langle x, z\rangle$ $\forall x \in V \Rightarrow y = z$.

(xxx) Define Norm of a vector.

(xxxi) if $u = (1, -1, 0)$ Then Compute $\|u\|$

(xxxii) Let V be an IPS over R,
Then for all $x, y \in V$ & $\alpha \in F$, Then prove that: $\|\alpha x\| = \alpha\|x\|$.

(xxxiii) Let V be an IPS over F, ~~State and prove~~
Then prove that: $|\langle x, y\rangle| \leq \|x\| \cdot \|y\|$
(Cauchy-Schwarz inequality) $x, y \in V$.

(xxxiv) $\|x+y\| \leq \|x\| + \|y\|$, $\forall x, y \in V$ (IPS)
(Triangle inequality)

(xxxv) What is the condition for ~~org~~ orthogonal vector.

(xxxvi) Define orthonormal vector and orthonormal set.

(xxxvii) Let $x = (2, 1+i, i)$ and $y = (2-i, 2, 1+2i)$ be vectors in $\mathbb{C}^3$. Compute $\langle x, y\rangle$, $\|x\|$, $\|y\|$, & $\|x+y\|$.
Then verify Cauchy Schwarz inequality & the triangle inequality.

(xxxviii) $V = C[0,1]$, let $f(t) = t$ & $g(t) = e^t$.
Compute $\langle f, g\rangle$, $\|f\|$, $\|g\|$ and $\|f+g\|$. Then verify both the Cauchy Schwarz inequality & the triangle inequality.

(xxxix) Let V be an IPS & x & y are orthogonal vector. Then Prove that $\|x+y\|^2 = \|x\|^2 + \|y\|^2$ $\forall x, y \in V$

(xxxx) Prove the Parallelogram Law on an IPS V.
i.e show that: $\|x+y\|^2 + \|x-y\|^2 = 2(\|x\|^2 + \|y\|^2)$ $\forall x, y \in V$.

(xxxxi) Define orthonormal basis for a V.S. V.

(xxxxii) Show that: The set $\left\{\left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right), \left(\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)\right\}$ is an orthonormal basis for R^2.

(xxxxiii) Let V be an IPS and $S = \{u_1, u_2, \ldots, u_k\}$ be an orthogonal subset of V consisting of non-zero vectors. If $y \in \text{span}(S)$ Then prove that
$y = \sum_{i=1}^{k} \frac{\langle y, u_i\rangle}{\|u_i\|^2} \cdot u_i$.

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(xxxxiv) Let V be an IPS and let S be an orthogonal subset of V consisting of non-zero vectors. Then prove that 'S' is L.I.

(xxxxv) Show that the ~~set~~ orthonormal set

$$\left\{ \frac{1}{\sqrt{2}}(1,1,0),\ \frac{1}{\sqrt{3}}(1,-1,1),\ \frac{1}{\sqrt{6}}(-1,1,2) \right\}$$ is

an orthonormal basis for R^3.

(xxxxvi) Every FDIPS possesses an orthonormal basis.

(xxxxvii) In R^4, Let $\omega_1 = (1,0,1,0)$, $\omega_2 = (1,1,1,1)$, $\omega_3 = (0,1,2,1)$

Then construct ~~compute~~ a orthonormal basis for R^4.